UNIVERSITIES OF MANCHESTER LIVERPOOL
LEEDS SHEFFIELD AND BIRMINGHAM

JOINT MATRICULATION BOARD

GENERAL CERTIFICATE OF EDUCATION

MATHEMATICS—Paper II

ADVANCED

Monday 14 June 1965 9.30-12.30

Careless and untidy work will be penalized.

Answer **seven** *questions.*

1. Two equal uniform rods *AB*. *AC* each of weight *W* and length 2a, and a third uniform rod of weight W_1, are freely hinged together to form a triangle *ABC* in which the angle *BAC* is 2θ. The triangle hangs in a vertical plane from a smooth pivot at *B,* and a couple is applied to the rod *AB* so as to keep the triangle in equilibrium with *BC* horizontal and *A* below *BC*. Find (i) the moment of the couple, showing its sense in a diagram, and (ii) the horizontal and vertical components of the forces exerted on *AC* by *BC* and *AB*.

2. A uniform rod AB of weight W and length $4a$ rests on a fixed cylinder of radius $a\sqrt{3}$ whose axis is horizontal. The rod is also horizontal, perpendicular to the axis of the cylinder, and the point of contact is C where AC is of length a. It is kept in this position by a light smooth string, attached to A, which rests against the surface of the cylinder and has a weight W_1 hanging at its other end. Prove the geometrical fact that at A the angle between the rod and the string is 120^0, and hence find the ratio of the weights W_1 and W.

Find the normal reaction and the frictional force at C, and prove that the coefficent of friction at this point must be greater than or equal to $\frac{1}{6}\sqrt{3}$

3. A uniform wire is bent to form a circular arc subtending an angle 2α at the centre. Prove that the distance of the centre of mass from the centre of the circle is $(r\sin\alpha)/\alpha$ where r is the radius.

A uniform wire in the form of a semicircle of radius r rests against a smooth vertical wall and is in a vertical plane perpendicular to the wall with its middle point in contact with the wall It is kept in equilibrium in this position by means of a light string of appropriate length joining a point B which divides the length of the wire in the ratio 1 : 3 to a point A on the wall. Explain how to fix the direction of the string and prove that it makes an angle θ with the horizontal where

$$\tan\theta = \frac{\pi}{\pi - 2\sqrt{2}}$$

If the weight of the wire is W, find the reaction at the point of contact with the wall.

4. A light inextensible string of length l is threaded through a smooth bead of mass m and has one end fixed at a point A on a smooth horizontal table and the other at a point B at a height $\frac{1}{2}l$ vertically above A. The bead is projected so as to describe a circle in contact with the table with angular velocity ω. Find the radius of the circle. Prove that the tension in the string is $\frac{15}{64}ml\omega^2$, and that ω must not exceed a certain value. Find this value.

5. A light elastic string, of natural length *a* and modulus *2mg,* is attached at one end to a fixed point *A* on a smooth horizontal table and at the other end to a particle *P* of mass 2*m*. A second light string which is *inextensible* is attached to *P* and has at its other end a second particle Q of mass *m.* The distance from *A* to the edge of the table is 2*a*. Initially the string *PQ* passes over the edge, the plane *APQ'* is perpendicular to the edge, and Q hangs below the edge. *P* is held on the table with the elastic string just taut and then released. Write down the equations of motion for *P* and Q when the distance *AP* is $\frac{3}{2}a + x$. Hence find the differential equation of the motion, and the time taken for *P* to reach the edge of the table.

Find also the tension in each string just as *P* reaches the edge of the table.

6. An engine of mass 56 tons moves from rest on a horizontal track against constant resistances, and for the first 30 seconds of its motion its acceleration is $\frac{2}{21}(30-t)$ ft. per sec^2 where *t* seconds is the time from the start. If the force exerted by the engine is initially $5\frac{1}{3}$ tons wt., find the magnitude of the resistances and an expression for the force exerted by the engine while *t* is less than 30, giving both answers in tons wt.

Calculate the speed of the engine after 21 seconds and the horsepower at which it is then working.

(Take g as 32 ft. per sec.)

7. An aircraft flying straight and level at constant speed passed over two landmarks *A* and *B,* where *B* is 540 miles N.E. from *A.* If the wind was blowing at 60 m.p.h. from due north and the speed of the aircraft in still air is 300 m.p.h. find the course on which it was flying and the time taken for the distance *AB*.

On the return journey the time taken for the same distance was 1 hr. 40 mins. If the speed of the wind was the same but its direction was from α W. of N., α being acute, find the value of α

8. A ring of mass *m* can slide on a smooth vertical rod and is attached to a light inextensible string which passes over a smooth peg at a distance *a* from the rod. At the other end of the string hangs a particle of mass *M.* The ring is released from rest at a point level with the peg and drops a distance *x* while the particle rises a distance *y*. Express y in terms of *x*, find dy/dt in terms of *x* and *dx/dt* and write the equation of energy for the system.

Given that $M(\sqrt{2}-1) < m < M$. prove that when $x = a$

$$\left(\frac{dx}{dt}\right)^2 = \frac{4ga[m - M(\sqrt{2}-1)]}{2m + M}$$

and that the system first comes to rest when

$$x = 2mMa/(M^2 - m^2)$$

9. Two equal particles are projected at the same instant from points *A* and *B* at the same level, the first from *A* towards *B* with velocity *u* at 45^0 above *AB*. and the second from *B* towards *A* with velocity *v* at 60^0 above *BA*. If the particles collide directly when each reaches its greatest height, find the ratio $v^2 : u^2$ and prove that $u^2 = ga(3-\sqrt{3})$, where *a* is the distance *AB*

After the collision the first particle falls vertically. Show that the coefficient of restitution between the particles is $(\sqrt{3}-1)/(\sqrt{3}+1)$.